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NOTES ON THE CALCULATION OF THE MINIMUM HORIZONTAL TAIL SURFACE FOR AIRPLANES EQUIPPED WITH WING FLAPS

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SUMMARY

A method of calculating the horizontal tail area for an airplane equipped with wing flaps is presented. The general problem of tail design, the effects of flaps on the factors involved, and the manner in which the flaps change the requirements upon which the minimum horizontal tail area is based are discussed.

INTRODUCTION

In connection with a flight investigation of different types of wing flaps it was observed that, where flaps had been installed on an airplane not designed for them, longitudinal instability in the form of a reversal of elevator control forces at low angles of attack usually occurred when the flaps were lowered. An increase in the horizontal tail area was generally required to make the airplane stable. A preliminary study to determine the amount that the horizontal tail surfaces should be increased to insure stability with the flapped wings indicated that existing methods of tail design, which had been formulated prior to the time wing flaps had come into general use, were not adequate for the purpose. A further study was therefore made to evolve a method of tail design applicable to airplanes with wing flaps. As a result of this study the method for the calculation of the minimum horizontal tail area discussed in this paper was developed. It combines and extends the two methods given by Diehl in reference 1 to obtain the tail area necessary for a statically stable airplane. In addition to the area required for stability, that required for longitudinal trim is considered. The method is applicable to any type of wing or high-lift device.

GENERAL MOMENT EQUATIONS

The complete equations of the moments acting on an airplane in flight are too involved for design use. Also, the information on some of the factors is so inadequate that the use of the complete equations is not justified at the present time. The use of the simplifying assumptions made in reference 1 has, therefore, been continued. So little is known of the effects of propeller thrust and slipstream that the discussion has been confined to the condition of power-off, or gliding, flight.

The moment M about the center of gravity of an airplane equals the sum of the wing moment M_w, the tail moment M_t, and the residual moment M_r attributable to the fuselage, the landing gear, and the exposed structural members. The residual moment is generally small relative to wing and tail moments and may be neglected. The total moment then becomes

$$M = M_w + M_t \tag{1}$$

By definition

$$M = C_m\ S\ c\ q \tag{2}$$

so that

$$C_m = \frac{M_w}{S\ c\ q} + \frac{M_t}{S\ c\ q} \tag{3}$$

The two terms on the right become (see fig. 1)

$$\left.\begin{aligned}
\frac{M_w}{Scq} &= C_{m_0} + (C_D \cos\alpha - C_L \sin\alpha)\frac{z}{c} + (C_L \cos\alpha + C_D \sin\alpha)\ \frac{x}{c} \\
\frac{M_t}{Scq} &= \frac{S_t c_t q_t}{Scq}\ \left(C_{m_{0_t}}\right) + \frac{S_t z_t q_t}{Scq}\left[C_{D_t}\ \cos(\alpha-\epsilon) - C_{L_t}\ \sin(\alpha-\epsilon)\right] \\
&\quad - \frac{S_t l q_t}{Scq}\left[C_{L_t}\ \cos\ (\alpha-\epsilon) + C_{D_t}\ \sin\ (\alpha-\epsilon)\right]
\end{aligned}\right\} \tag{4}$$

where x is the distance of the center of gravity back of the aerodynamic center of the wing measured parallel to the thrust axis.

z, the distance of the center of gravity below the

aerodynamic center of the wing measured perpendicular to the thrust axis.

l, the distance, measured parallel to the thrust axis, of the center of gravity forward of the aerodynamic center of the horizontal tail surfaces usually taken as the elevator hinge axis.

z_t, the distance, measured perpendicular to the thrust axis, of the center of gravity below the aerodynamic center of the tail surfaces.

The angle of attack α is measured relative to the thrust axis, in radians, the subscript t refers to the horizontal tail surfaces, and the other terms have their usual significance.

In order to simplify the equations the following assumptions are made:

(a) All terms in the tail-moment equation are neglected except

$$\frac{S_t l q_t}{Scq}\left[C_{L_t} \cos (\alpha - \epsilon)\right]$$

(b) In the wing-moment equation the term $C_D \sin \alpha \frac{x}{c}$ is neglected.

(c) Sin α is considered equal to α and $\cos \alpha$ and $\cos (\alpha - \epsilon)$ equal to 1.

(d) The ratio q_t/q is treated as a constant.

(e) C_D is rewritten $C_{D_o} + \frac{C_L^2}{\pi A}$ where A is the effective aspect ratio of the wing.

On the basis of these assumptions, equation (3) may be rewritten

$$C_m = C_{m_o} + \left[C_{D_o} + \frac{C_L^2}{\pi A} - C_L \alpha\right]\frac{z}{c} + C_L \frac{x}{c} - C_{L_t} \frac{S_t l}{S c} \frac{q_t}{q} \quad (5)$$

The slope of the pitching-moment-coefficient curve obtained by differentiating equation (5) is

$$\frac{dC_m}{d\alpha} = \left[\frac{2C_L}{\pi A}\frac{dC_L}{d\alpha} - \alpha\frac{dC_L}{d\alpha} - C_L\right]\frac{z}{c} + \frac{dC_L}{d\alpha}\frac{x}{c} - \frac{dC_{L_t}}{d\alpha_t}\frac{d\alpha_t}{d\alpha}\frac{S_t l}{Sc}\frac{q_t}{q} \qquad (6)$$

Equation (6) shows that, where z is other than zero, the slope varies with the angle of attack. The most convenient way to deal with this variation of slope is by means of the second derivative

$$\frac{d\left(\frac{dC_m}{d\alpha}\right)}{d\alpha} = \frac{d^2C_m}{d\alpha^2} = \left[\frac{2}{\pi A}\left(\frac{dC_L}{d\alpha}\right)^2 - 2\frac{dC_L}{d\alpha}\right]\frac{z}{c} \qquad (7)$$

which is independent of angle of attack.

The values for the wing and tail characteristics used in the equations should include the interference effects caused by the presence of the fuselage. Reference 2 gives the most complete wing-fuselage-interference data available that would be useful in making an allowance for the interference effects. In the case of biplanes the characteristics of the complete cellule are used. Before being used for moment computations, the tail-surface characteristics are generally multiplied by a tail-efficiency factor η_t, usually 0.8, to account for fuselage interference. The tail-surface coefficients C_{L_t} and $dC_{L_t}/d\alpha_t$ may be either for the elevator-free or the elevator-fixed condition, depending on which type of stability is being investigated. As the elevator-free stability determines the minimum horizontal tail area, the following discussion is confined to this type of stability and the tail coefficients refer to the elevator-free condition unless otherwise noted. It should be appreciated that the elevator-free tail-surface characteristics depend to some extent on the weight moment of the elevator-control system about the elevator hinge. This moment is a variable dependent on the attitude of the airplane, and its inclusion greatly complicates the problem of design. Further, it generally tends to increase the stability of the airplane over that obtained with a weightless elevator. For these reasons the effects of elevator weight are neglected.

APPLICATION OF THE GENERAL MOMENT EQUATIONS TO THE DETERMINATION OF THE MINIMUM HORIZONTAL TAIL AREA

The primary function of the horizontal tail surfaces is to provide a means by which the pilot can control the angle of attack and the speed of the airplane. The design problem, therefore, consists of the determination of the tail area and the corresponding center-of-gravity position by which, with a reasonable tail length, the desired control will be obtained and the airplane will be stable. As the requirements for control and stability are different, they must be separately considered and the consideration that gives the larger tail area defines the minimum that may be used on the airplane. Although the control is the primary function of the tail surfaces, it is more convenient to deal first with the stability requirements.

Stability Requirements

It is generally conceded that airplanes should be statically stable. The need for complete dynamic stability is somewhat open to question for, as shown in reference 3, dynamic instability in several airplanes with which stability tests were made was not noted by the pilots until the phenomenon was specifically looked for. It is believed, however, that dynamic stability is desirable throughout the range of speeds in which airplanes may be flown with elevator free. Dynamic stability at speeds that must be held by the pilot is not considered important.

For static stability certain conditions relative to elevator travel and stick force are required. The trailing edge of the elevator should be moved progressively downward as the angle of attack is decreased; at all angles of attack above that at which the airplane is trimmed a pull should be required on the control column; at all angles below that for trim a push should be required. The first condition imposes the requirement that, with the elevator fixed, $dC_m/d\alpha$ be negative at all angles of attack, whereas the requirements regarding stick force imply that with the elevator free C_m be negative at all angles of attack above that for trim and positive at all angles below it. The requirements for dynamic stability with elevator free are that, within the range of the trimming angles of attack, $dC_m/d\alpha$ with the elevator free be nega-

tive and have a value that falls outside a limited range of values, which are defined by the amount of damping in pitching obtained from the horizontal tail surfaces and by physical dimensions and aerodynamic characteristics of the complete airplane not directly related to the tail length or area.

It can be shown that if $dC_m/d\alpha$ with the elevator free is negative, as is required for dynamic stability, $dC_m/d\alpha$ will also be negative with the elevator fixed so that it is possible to base the design of the tail surface on the elevator-free requirements with only incidental reference to the elevator-fixed condition. Examination of equation (6) shows that the difference between the values of $dC_m/d\alpha$ for the elevator-fixed and elevator-free conditions depends only on the values of $dC_{L_t}/d\alpha_t$ for the two conditions and their influence on the last term of the equation. As this term is essentially negative and the value of $dC_{L_t}/d\alpha_t$ is always less with the elevator free than fixed, the static stability will always be greater for the elevator-fixed condition. Therefore, if the airplane is designed with a negative value of $dC_m/d\alpha$ for the elevator-free condition, static stability for the elevator-fixed condition will be insured. Because in the development of the following design criterions the values of $dC_m/d\alpha$ with the elevator free are considered only for the range of stick-free trimming angles, however, two checks are required to insure stability with the elevator fixed over the entire flight range. These checks will be noted as the development proceeds.

The selection of the degree of static stability desired, that is, a suitable value of $dC_m/d\alpha$ for the elevator free to be used in design, is beyond the province of this paper. It might be first taken as the minimum that would insure dynamic stability with the elevator free. A compromise could be made if a preliminary design of the tail surfaces showed the stick forces to be objectionably large.

Data on the dynamic stability of airplanes for the elevator-free condition not being available, it seems best that an empirical value of $dC_m/d\alpha$ based on some existing satisfactory airplanes be used for the present. Diehl (reference 1) gives such data for the elevator-fixed con-

dition. From an inspection of the data with an allowance for the difference between the fixed- and free-elevator conditions, it would appear that a value of $dC_m/d\alpha = -0.1$ for the elevator-free condition at an angle of attack corresponding to a lift coefficient of 0.5 would probably give satisfactory stability characteristics. A rough check of the dynamic stability may be made by use of the charts of reference 4, which were, however, prepared for the elevator-fixed condition and therefore do no more than approximate the elevator-free condition.

The value of $dC_m/d\alpha$ at angles of attack other than that corresponding to a lift coefficient of 0.5 will depend on the vertical position of the center of gravity, z/c. If z/c is zero, the value of $dC_m/d\alpha$ will, of course, be constant over the entire range of angle of attack. In this paper it will be assumed that the value of z/c is based on other than stability considerations and is known prior to the designing of the tail. This assumption is in accordance with usual practice. The low-wing monoplane, for example, has come into widespread use on account of its adaptability to relatively simple wheel-retracting mechanisms, although the arrangement makes it difficult to obtain adequate stability at high angles of attack without making the airplane unduly stiff at low angles. As the change of $dC_m/d\alpha$ with angle is independent of the tail-surface design (equation (7)), the value of $dC_m/d\alpha$ at any angle of attack α may be computed immediately by means of the following equation:

$$\left(\frac{dC_m}{d\alpha}\right)_\alpha = \left(\frac{dC_m}{d\alpha}\right)_{\alpha_o} + \frac{d^2 C_m}{d\alpha^2} (\alpha_o - \alpha) \tag{8}$$

where α_o is the angle of attack corresponding to a lift coefficient of 0.5.

If the resulting value of $dC_m/d\alpha$ is positive at any angle within the elevator-free trimming range, the value of $dC_m/d\alpha$ at α_o should be increased until only negative values are obtained.

The horizontal tail area, as will be shown, depends on the range of angles of attack through which the airplane is to be flown and the angles of attack at which the airplane may be trimmed, elevator free. Both the complete flying range and the trimming range depend on the

purpose of the airplane and will be set by the designer. It may be desirable for an acrobatic airplane to be capable of flight at any angle between the positive and negative stalls, whereas a transport might be limited to an angle slightly less than that corresponding to the maximum speed.

For the elevator-fixed condition, the slope of the curve of pitching-moment coefficient is independent of the position of the elevator, stabilizer, or trimming tab. So long as the tail plane is not stalled, a readjustment of either of these surfaces results in a constant change of ordinate over the entire length of the curve. For the elevator-free condition, however, this situation does not necessarily hold. The airplane may be designed to have curves of pitching-moment coefficients of either of the types represented in figure 2. The case illustrated in figure 2(b), where the angle at which the curves change slope depends on the setting of the trimming device, will give the smaller tail area.

In either case a problem in tail design is to determine the value of the minimum tail area that will give the conditions desired. For the type of pitching-moment curve shown in figure 2(a), the tail should be of sufficient area that when the trimming device is set for the tail-heavy trimming angle (α_{T_1}) and the airplane is rotated in pitch to the minimum flying angle α_2, the negative angle of attack of the tail surface is not greater than that at which $dC_{L_t}/d\alpha_t$ becomes zero. If this angle is exceeded by a limited amount, the second type of curve will be obtained. Similarly with the stabilizer or trimming tab set for nose-heavy trim, the positive angle of attack of the tail surface should not be exceeded when the airplane is rotated to its stalling angle. If this type of C_m curve is chosen, the coefficient with tail-heavy trim at the minimum angle of attack will be

$$C_{m_{\alpha_2}} = \left(\frac{dC_m}{d\alpha}\right)_{T_1} (\alpha_2 - \alpha_{T_1}) - \frac{d^2C_m}{d\alpha^2} \frac{(\alpha_2 - \alpha_{T_1})^2}{2} \qquad (9)$$

With nose-heavy trim, the moment coefficient at the maximum angle of attack will be

$$C_{m_{\alpha_1}} = \left(\frac{dC_m}{d\alpha}\right)_{\alpha_{T_2}} (\alpha_1 - \alpha_{T_2}) + \frac{d^2C_m}{d\alpha^2} \frac{(\alpha_1 - \alpha_{T_2})^2}{2} \qquad (10)$$

Usually $C_{m_{\alpha_2}}$ is larger than $C_{m_{\alpha_1}}$, so that only $C_{m_{\alpha_2}}$ will be considered. The treatment of $C_{m_{\alpha_1}}$ is identical with that given for $C_{m_{\alpha_2}}$. It should be appreciated that equations (9) and (10) may be evaluated without consideration of the horizontal tail surface on the basis of the preceding discussion.

The value of $C_{m_{\alpha_2}}$ given by equation (9) is then substituted into equation (5) giving

$$\left(\frac{dC_m}{d\alpha}\right)_{\alpha_{T_1}} (\alpha_2 - \alpha_{T_1}) - \frac{d^2 C_m}{d\alpha^2} \frac{(\alpha_2 - \alpha_{T_1})^2}{2} = C_{m_o}$$

$$+ \left[C_{D_o} + \frac{{C_{L_{\alpha_2}}}^2}{\pi A} - C_{L_{\alpha_2}} \alpha_2 \right] \frac{z}{c} + C_{L_{\alpha_2}} \frac{x}{c} - C_{L_t} \frac{S_t l}{Sc} \frac{q_t}{q} \qquad (11)$$

In equation (11) the unknowns are x/c, C_{L_t}, and the term $S_t l/Sc$, which will be designated the "tail-volume coefficient." As the ratio $S_t l/Sc$ will vary inversely with C_{L_t}, it is obvious that the smaller value of the tail volume will be obtained when C_{L_t} is placed equal to $C_{L_{t_{max}}}$.

From equation (6)

$$\left(\frac{dC_m}{d\alpha}\right)_{\alpha_{T_1}} = \left[\frac{2C_{L_{\alpha_{T_1}}}}{\pi A} \frac{dC_L}{d\alpha} - \alpha_{T_1} \frac{dC_L}{d\alpha} - C_{L_{\alpha_{T_1}}} \right] \frac{z}{c} + \frac{dC_L}{d\alpha} \frac{x}{c}$$

$$- \frac{dC_{L_t}}{d\alpha_t} \frac{d\alpha_t}{d\alpha} \frac{S_t l}{Sc} \frac{q_t}{q} \qquad (12)$$

The unknowns are x/c, $d\alpha_t/d\alpha$, and $S_t l/Sc$. The value of $d\alpha_t/d\alpha$ depends on l but, on the assumption of a conventional tail length, the designer may assign an approximate value, which may later be checked if the actual l varies greatly from the assumed value. As both equations (11) and (12) contain the same two unknowns, the required loca-

tion of the center of gravity and the tail-volume coefficient for the stated conditions may be obtained by simultaneous solution.

For the elevator-free condition, no consideration need be given to the values of $dC_m/d\alpha$ outside the elevator-free trimming range. In order to meet the requirement for elevator-fixed stability, however, the values of $dC_m/d\alpha$ with the elevator fixed should be negative at all angles between α_1 and α_2. As instability will appear first at the limiting angles, the values of $dC_m/d\alpha$ at α_1 and α_2 for the elevator-fixed condition should be computed on the basis of the values of $S_t l/Sc$ and x/c, obtained from the foregoing equations, to ascertain that the requirement has been met.

If a smaller tail volume than that given by equations (11) and (12) is employed, the setting of the stabilizer required for trim at α_{T_1} will be such that, when the airplane is rotated in pitch to α_2, the angle of attack of the tail surface will increase negatively to an angle greater than that at which $dC_{L_t}/d\alpha$ becomes zero. If the tail-surface characteristics abruptly changed and burbling set in at the angle of attack at which $dC_{L_t}/d\alpha_t$ becomes zero, a reduction of tail surface below that given by equations (11) and (12) would be undesirable. Some information on the subject of elevator-free tail-surface characteristics obtained from references 5 and 6 is given in figure 3. The indications are that with the elevator free the tail surface reaches its maximum lift coefficient at an angle of attack well below that of the undeformed section. The lift coefficient is then maintained practically constant, at least to the stalling angle of the undeformed section. With tail surfaces having lift curves of this type, illustrated by the solid curve of figure 3, it is possible to reduce with safety the tail volume below that given by equations (11) and (12). When the tail volume is thus reduced, a curve of pitching-moment coefficient of the type illustrated in figure 2(b) will be obtained.

The limiting condition for this type of pitching-moment curve is reached when $C_{m_{\alpha_2}}$ approaches zero. Reference to equation (5) will show that this condition is reached when the tail moment approaches the wing moment in

value. This condition may, therefore, be represented by

$$C_{L_{t_{max}}} \frac{S_t l}{Sc} \frac{q_t}{q} > C_{m_o} + \left[C_{D_o} + \frac{{C_{L_{\alpha_2}}}^2}{\pi A} - C_{L_{\alpha_2}} \alpha_2 \right] \frac{z}{c} + C_{L_{\alpha_2}} \frac{x}{c} \tag{13}$$

For the determination of the location of the center of gravity and the tail-volume coefficient required for this condition, the inequality sign may be replaced by an equality sign and the equation solved simultaneously with equation (12). The tail volume obtained should be arbitrarily increased by a small amount in order to insure inequality in equation (13).

When equation (13) is used as a basis for the determination of the tail size, it is necessary to make an additional computation to determine that with the airplane trimmed at α_{T_1} the tail surface will not be stalled when the airplane is flown at α_2. If the setting of the stabilizer required for trim at α_{T_1} is such that the surface will stall at α_2, the value of $dC_m/d\alpha$ for the elevator-fixed condition will not be negative as required. The computation is based on the following considerations.

The minimum angle of attack that the tail can have at α_2 is the angle that corresponds to the angle where $dC_{L_t}/d\alpha_t$ with the elevator free becomes zero. In this case the airplane will have the pitching-moment curve illustrated by the dotted curve of figure 2(b), and will trim at an angle of attack only slightly above α_2. In order to obtain a condition of trim at α_{T_1} it is necessary to adjust the stabilizer nose downward through an angle great enough to cause a change in tail moment equal to the ordinate of the dotted curve at α_{T_1}. As the dotted curve will have the same value of $dC_m/d\alpha$ at α_{T_1} as the curve that gives trim at that angle, the ordinate will be given by the following equation

$$C_{m_{\alpha_{T_1}}} = C_{m_{\alpha_2}} - \left[\left(\frac{dC_m}{d\alpha}\right)_{\alpha_{T_1}} (\alpha_2 - \alpha_{T_1}) - \frac{d^2C_m}{d\alpha^2} \frac{(\alpha_2 - \alpha_{T_1})^2}{2} \right] \tag{14}$$

where $C_{m_{\alpha_2}}$ is the residual moment at α_2 after the con-

ditions imposed by equation (13) have been satisfied. In order to meet the requirement, the difference between the stalling angle of the tail surface and the angle at which $dC_{L_t}/d\alpha_t$ becomes zero must be greater than the angular change of the stabilizer setting required to give this change of moment. If the difference between the stalling angle of the tail surface and the angle at which $dC_{L_t}/d\alpha_t$ becomes zero is designated by $\Delta\alpha_t$, the requirement can be represented by the following expression

$$\Delta\alpha_t \frac{dC_{L_t}}{d\alpha_t} \frac{S_t l}{Sc} \frac{q_t}{q} > -C_{m_{\alpha_{T_1}}} > \left(\frac{dC_m}{d\alpha}\right)_{\alpha_{T_1}} (\alpha_2 - \alpha_{T_1}) - \frac{d^2 C_m}{d\alpha^2} \frac{(\alpha_2 - \alpha_{T_1})^2}{2} - C_{m_{\alpha_2}} \qquad (15)$$

This equation may be treated similarly to equation (13). If the tail volume given by this equation is larger than that given by equation (13), it should be used as the basis of further design. In this case the location of the center of gravity obtained simultaneously should also be used.

Balance Requirements

In the preceding discussion the minimum tail volume has been determined on the basis of stability. No smaller volume than that given on this basis may be used. In order to obtain the desired speed range, however, a larger tail volume may be required. The volume required for balance at various speeds will now be considered. For steady flight at α_2, the pitching-moment coefficient must be zero. The elevator must be deflected from its free-floating position through a sufficient angle to reduce the tail moment by an amount equal to the pitching moment of the airplane with the elevator free. That is,

$$\Delta C_{L_t} \left(\frac{S_t l}{Sc}\right) \frac{q_t}{q} = C_{m_{\alpha_2}} \qquad (16)$$

where ΔC_{L_t} is the change of tail lift coefficient caused by deflecting the elevator from its free-floating position. For the type of stability curve shown in figure 2(b) the pitching moment is larger at other angles between α_{T_1}

and α_2 than at α_2. In this case it is essential that equation (16) hold when the maximum pitching moment is substituted for the moment at α_2.

The value of ΔC_{L_t} will be a maximum when the elevator is deflected in the order of 60° relative to the stabilizer. For various practical reasons, the most important of which is that the pilot will probably not be able to exert sufficient force to deflect the elevator through this angle, the elevator travel is usually limited to a range from ±20° to ±25°. The values of ΔC_{L_t} for one tail plan form and section for different elevator-tail surface ratios obtained from reference 6 are plotted in figure 4 for two different stabilizer angles. Particular note should be made of the fact that ΔC_{L_t} decreases with the angle of attack of the stabilizer. From this fact arises the necessity of checking the control between maximum C_m and $C_{m_{\alpha_2}}$ for the type of pitching-moment curve shown in figure 2(b).

A comparison of the two bases for the determination of the horizontal tail volume shows that the volume required to obtain the desired speed range with a given tail arrangement depends only on the airplane pitching-moment coefficient at α_2. It is independent of the wing characteristics. The volume required for stability is also dependent on $C_{m_{\alpha_2}}$ but is, in addition, dependent on several factors (C_{m_0}, C_{D_0}, and $C_{L_{\alpha_2}}$) that are functions of the wing section used. If the summation of the terms involving these factors is zero, a condition readily accomplished with plain wings, both the balance and stability requirements will depend on the same factor, $C_{m_{\alpha_2}}$. A comparison of figures 3 and 4 indicates that for the tail arrangements given for c_e/c_t ratios of the order of 0.45, $C_{L_{t_{max}}}$ and ΔC_{L_t} are equal. For smaller ratios $C_{L_{t_{max}}}$ is larger, becoming twice ΔC_{L_t} for $\frac{c_e}{c_t} = 0.25$. For ratios below 0.45, the tail volume would therefore be defined by equation (16).

As the terms depending on the wing characteristics increase from zero, the volume given by the stability equations would first approach and then exceed that given by the balance equation. In the range below that in which

the two requirements give the same value, the tail volume would be constant regardless of the wing section employed. In the range above, the tail volume would vary directly with the wing factors C_{m_o} and C_{D_o}.

The Effect of Flaps on the Tail Volume

In addition to increasing the lift of a wing, wing flaps increase the pitching-moment coefficient C_{m_o} and usually, the profile-drag coefficient C_{D_o}. As with the lift, the amounts by which the pitching-moment and drag coefficients are increased depend on the size of the flap. For effective flap installations the increases are large and greatly influence the tail area required for stability. As high control forces are associated with large tail surfaces, the increases caused by the use of flaps are undesirable. A means of avoiding part of the increase has already been suggested; that is, to design for a pitching-moment curve of the type illustrated in figure 2(b).

As flapped airplanes are not intended for inverted flight with flaps extended, a simple approximation of the minimum tail volume may be obtained by choosing α_2 to correspond to the angle of attack at which the lift is zero. For this angle equation (13) reduces to

$$C_{L_{t_{max}}} \left(\frac{S_t l}{Sc}\right) \frac{q_t}{q} > C_{m_o} + C_{D_o} \frac{z}{c} \qquad (17)$$

CONTROL-FORCE CONSIDERATIONS

An airplane may have adequate tail volume for stability and balance and still not be satisfactory because the stick forces required to operate the elevators may be in excess of the force that the pilot can comfortably apply. The present knowledge of comfortable elevator forces, however, is not sufficient for design purposes. The elevator-force requirements are therefore usually specified as an arbitrary maximum that for static conditions should never be exceeded. As the elevator force increases with speed, it is usually necessary to investigate conditions only at the high-speed end of the flying range. Where α_2 includes the negative-lift angles, the elevator forces should be investigated for diving conditions at zero lift. For the stability and balance requirements, it is possible

to deal with nondimensional equations, whereas with elevator force it is necessary to make the equations dimensional.

The elevator force, which is defined as the force applied to the top of the control column, is first converted to a moment about the elevator hinge. It should be appreciated that with the conventional limitations of the elevator deflection and the standardization of the movement of the top of the control columns, all elevators have approximately the same mechanical advantage relative to the control column. It is treated, therefore, as a constant. The hinge moment may then be written

$$H = C_h \; K \; S_t^{3/2} \; q_t \tag{18}$$

where $K = \dfrac{S_e c_e}{S_t^{3/2}}$. Up to a value of approximately 0.1, the hinge-moment coefficient C_h may be written

$$C_h = \frac{dC_h}{dC_{L_t}} \, \Delta C_{L_t} \tag{19}$$

where ΔC_{L_t}, in this case, is equal to

$$\Delta C_{L_t} = \frac{C_m}{\left(\dfrac{S_t l}{Sc}\right) \dfrac{q_t}{q}} \tag{20}$$

where C_m is assigned the value of the pitching-moment coefficient at the minimum angle of attack at which the airplane is to be flown with a given setting of the trimming device, unless the angle of attack is below that for zero lift. In such a case C_m is assigned the value for zero lift. dC_h/dC_{L_t} depends on the c_e/c_t ratio chosen.

When the values from equations (19) and (20) are substituted in equation (18), the hinge moment becomes

$$H = \frac{dC_h}{dC_{L_t}} \, \frac{C_m}{\left(\dfrac{S_t l}{Sc}\right)} \, K \; S_t^{3/2} \; q \tag{21}$$

where q is the dynamic pressure corresponding to the

conditions for which C_m was taken. The equation may be solved for the required S_t to give the limiting hinge moment

$$S_t^{3/2} = \frac{H\left(\frac{S_t l}{Sc}\right)}{K \frac{dC_h}{dC_{L_t}} C_m q} \tag{22}$$

From the value of S_t obtained and the tail volume $\left(\frac{S_t l}{Sc}\right)$, the tail length l may be computed. If it is unreasonably large, means of changing $\frac{dC_h}{dC_{L_t}}$ either by changing the $\frac{c_e}{c_t}$ ratio or the aspect ratio (which would require a complete recomputation of the tail volume and area) or by aerodynamic balancing should be considered. It should be appreciated that equation (22) holds good only up to $C_h = 0.1$.

THE EFFECT OF THE c_e/c_t RATIO ON THE REQUIRED TAIL AREA

The effect of the c_e/c_t ratio on the tail factors $C_{L_{t_{max}}}$, ΔC_{L_t}, and $K \frac{dC_h}{dC_{L_t}}$ for a given plan form is illustrated by figures 3, 4, and 5, respectively, which show that $C_{L_{t_{max}}}$ and $K \left(\frac{dC_h}{dC_{L_t}}\right)$ decrease while ΔC_{L_t} increases with increases in the ratio. As the tail area varies inversely as the factors, it is evident that the ratio should be made as small as possible compatible with a ΔC_{L_t} large enough to give adequate control. If the elevator forces work out too large for this combination, recourse should be had to aerodynamic balancing.

The possibility of using larger elevator deflections to increase ΔC_{L_t} naturally suggests itself. The procedure, however, is not recommended because, as shown by figure 5, dC_h/dC_{L_t} rapidly increases in the neighborhood of a 20° elevator deflection, so that the hinge moment will rapidly increase for only slight increases in ΔC_{L_t}.

Also, the mechanical advantage of the control column over the elevator will also decrease for larger elevator deflections.

COMPARISON BETWEEN ADJUSTABLE STABILIZER AND TRIMMING TABS

With an adjustable stabilizer a change of trim is accomplished by changing the orientation of the stabilizer relative to the fuselage. No change of tail-plane characteristics is involved and the surface may be directly designed on the basis of the preceding discussion. Trimming tabs, however, change the trim by changing the angle at which the elevator floats relative to the stabilizer. The change of floating angle is accompanied by a change in the tail-plane characteristics $C_{L_{t_{max}}}$, ΔC_{L_t}, and dC_h/dC_{L_t}. Both $C_{L_{t_{max}}}$ and ΔC_{L_t} are increased as the trimming tab is deflected downward for tail-heavy trim and decreased for the upward deflection corresponding to nose-heavy trim. For an airplane having a curve of pitching-moment coefficient of the type illustrated in figure 2(a), the use of a trimming tab instead of an adjustable stabilizer will result in a smaller tail surface because of the higher values of ΔC_{L_t} and $C_{L_{t_{max}}}$ when the airplane is trimmed at α_{T_1}. This consideration suggests the possibility of building all elevators with trailing edges reflexed downward to increase $C_{L_{t_{max}}}$ and ΔC_{L_t}. With the type of curve illustrated in figure 2(b), it is desirable to check the stability with nose-heavy trim because the decrease in $C_{L_{t_{max}}}$ may be of sufficient magnitude to cause the airplane to become unstable at α_2.

The effect of the tab setting on $\frac{dC_h}{dC_{L_t}}$, as indicated by figure 6 (derived from the data of reference 7), is appreciable. A deflection of the tab from neutral in either direction increases $\frac{dC_h}{dC_{L_t}}$ so that the variation of elevator force with speed will be greater at the extreme than at the intermediate trimming angles.

The amount by which the tail lift coefficient may be

varied at a given angle of attack of the stabilizer depends on the size and deflection of the tab. A check of the sufficiency of a proposed tab installation may be obtained by the following equation

$$\Delta C_{L_{t(tab)}} \geqq \frac{\left(\frac{dC_m}{d\alpha}\right)_{\alpha_{T_2}} (\alpha_{T_1} - \alpha_{T_2}) + \frac{d^2 C_m}{d\alpha^2} \frac{(\alpha_{T_1} - \alpha_{T_2})^2}{2}}{\frac{S_t l}{Sc} \frac{q_t}{q}} \tag{22}$$

where $\Delta C_{L_{t(tab)}}$ is the amount by which the elevator-free lift coefficient may be varied by means of the trimming tabs at a constant angle of attack.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., March 15, 1937.

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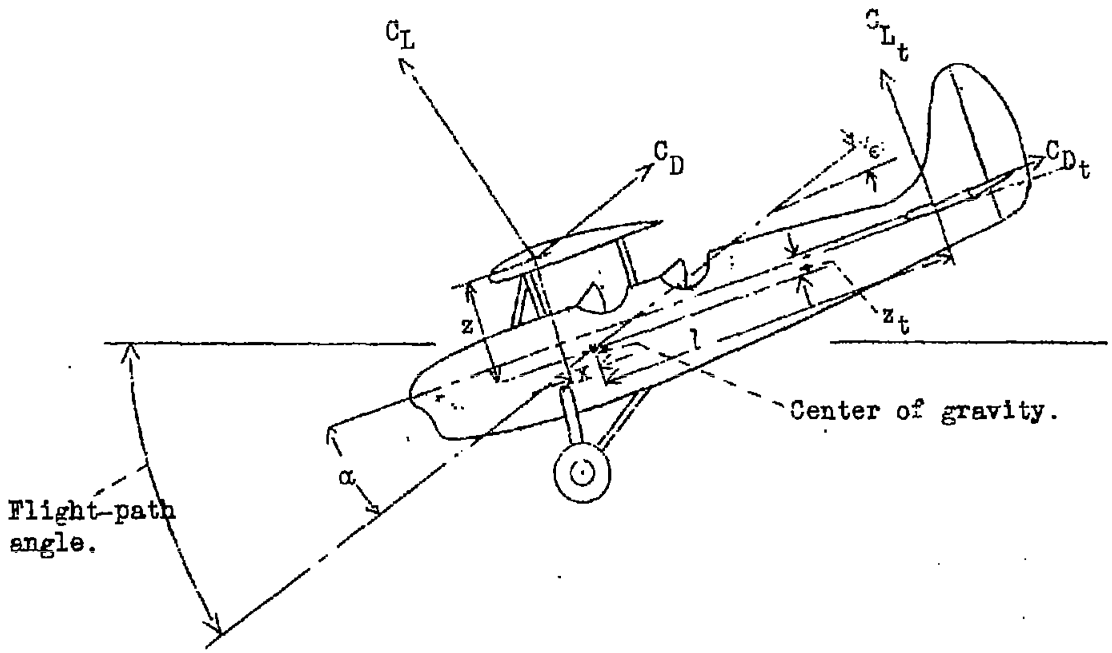
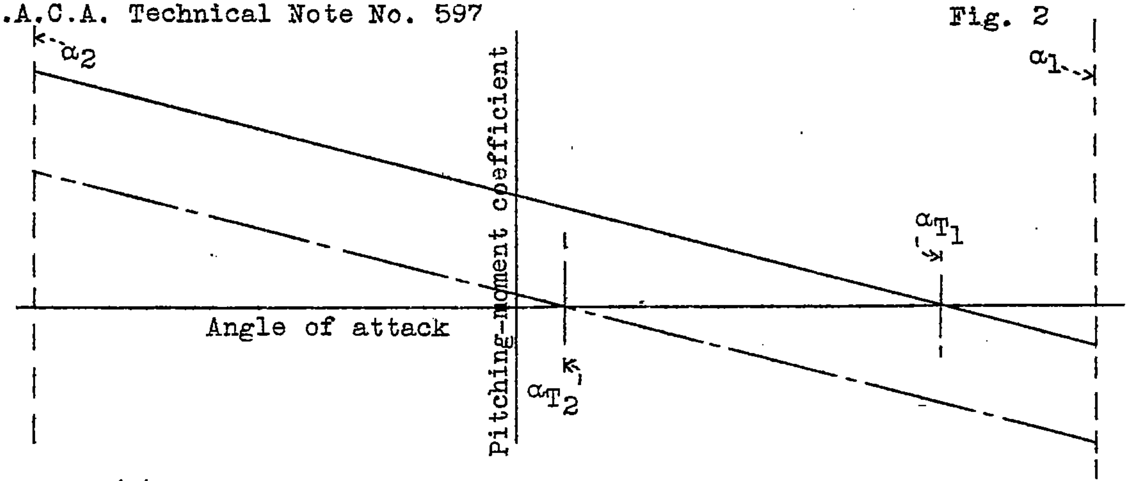
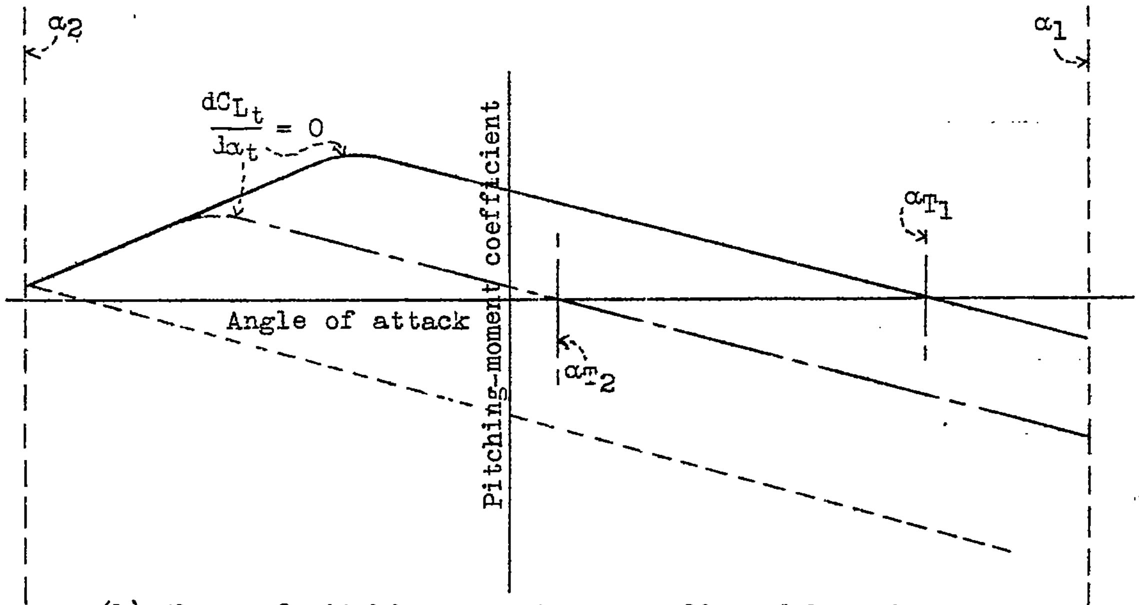


Figure 1.- Layout of wing and tail forces.



(a) Shape of pitching-moment curve independent of trim.



(b) Shape of pitching-moment curve affected by trim.

Figure 2.- Possible types of pitching-moment curve.

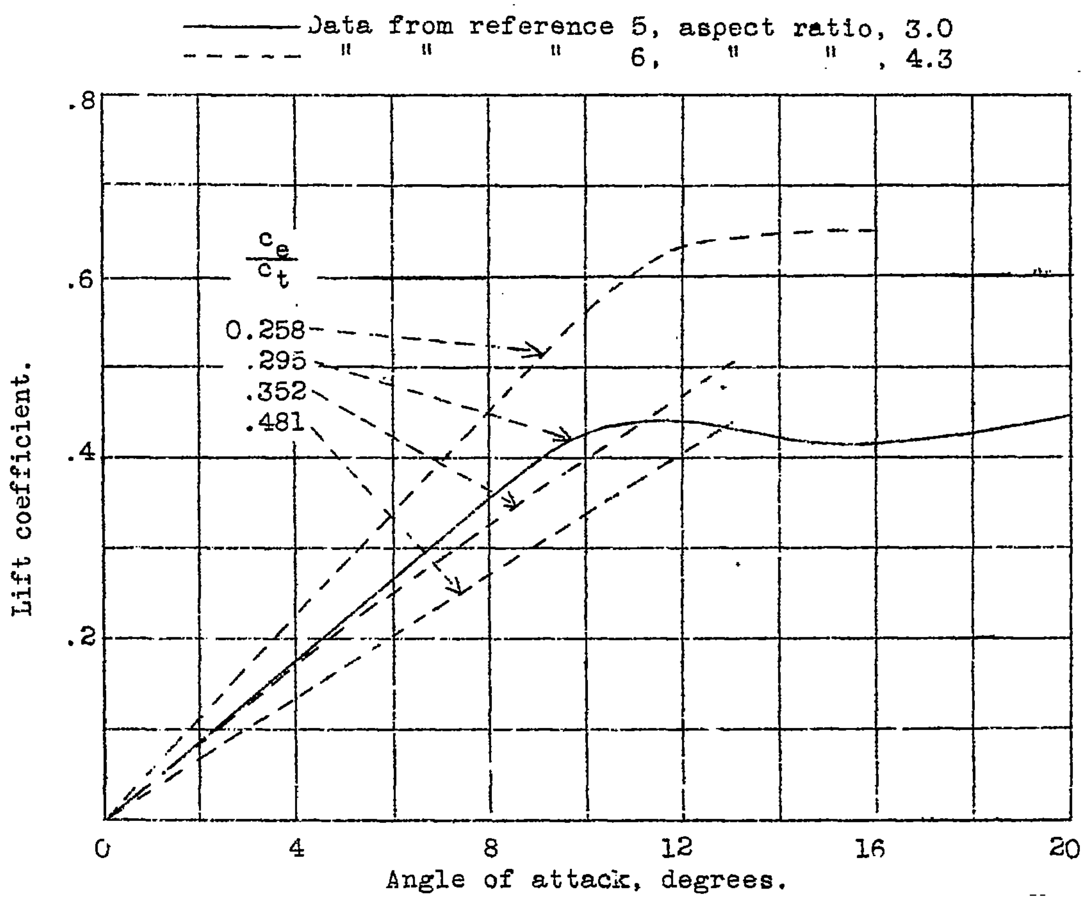


Figure 3.- Typical tail-surface elevator-free lift curves (references 5 and 6).

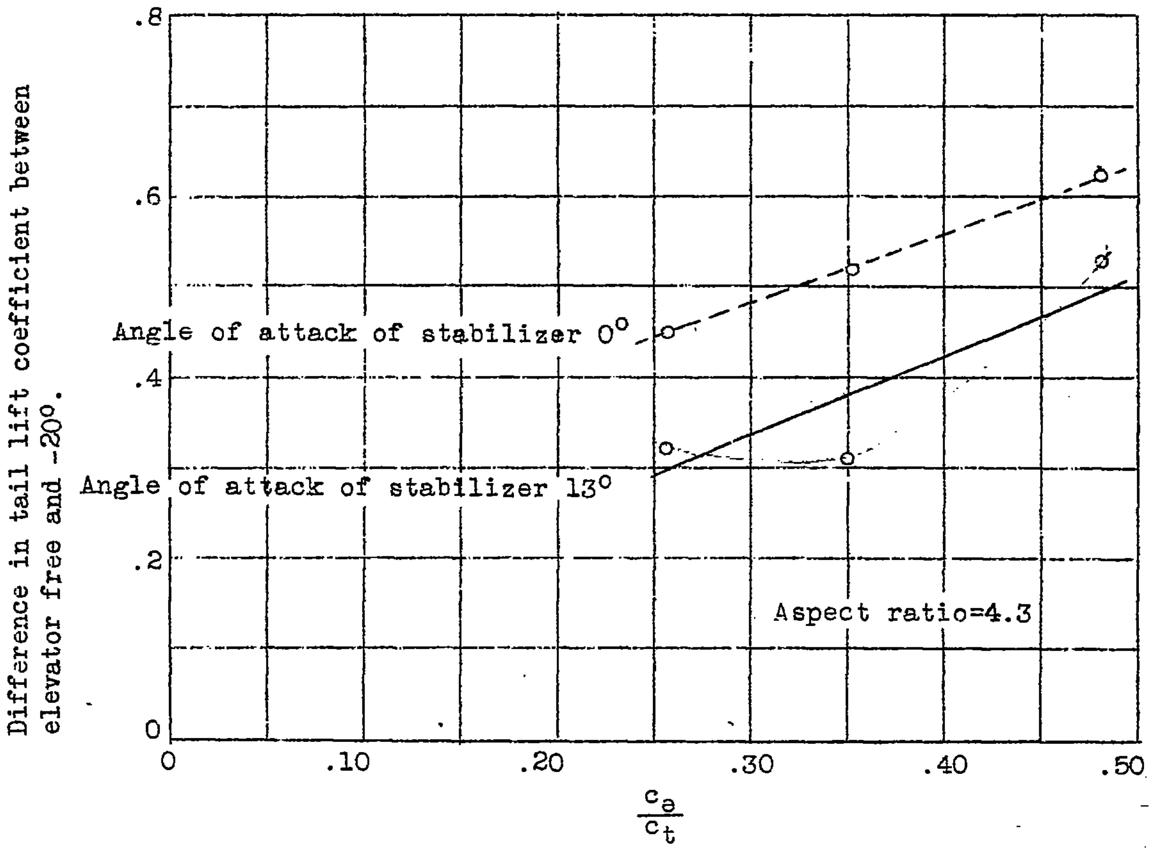


Figure 4.- Effect of ratio of elevator chord to total tail-surface chord on lift change possible with elevator (reference 6).

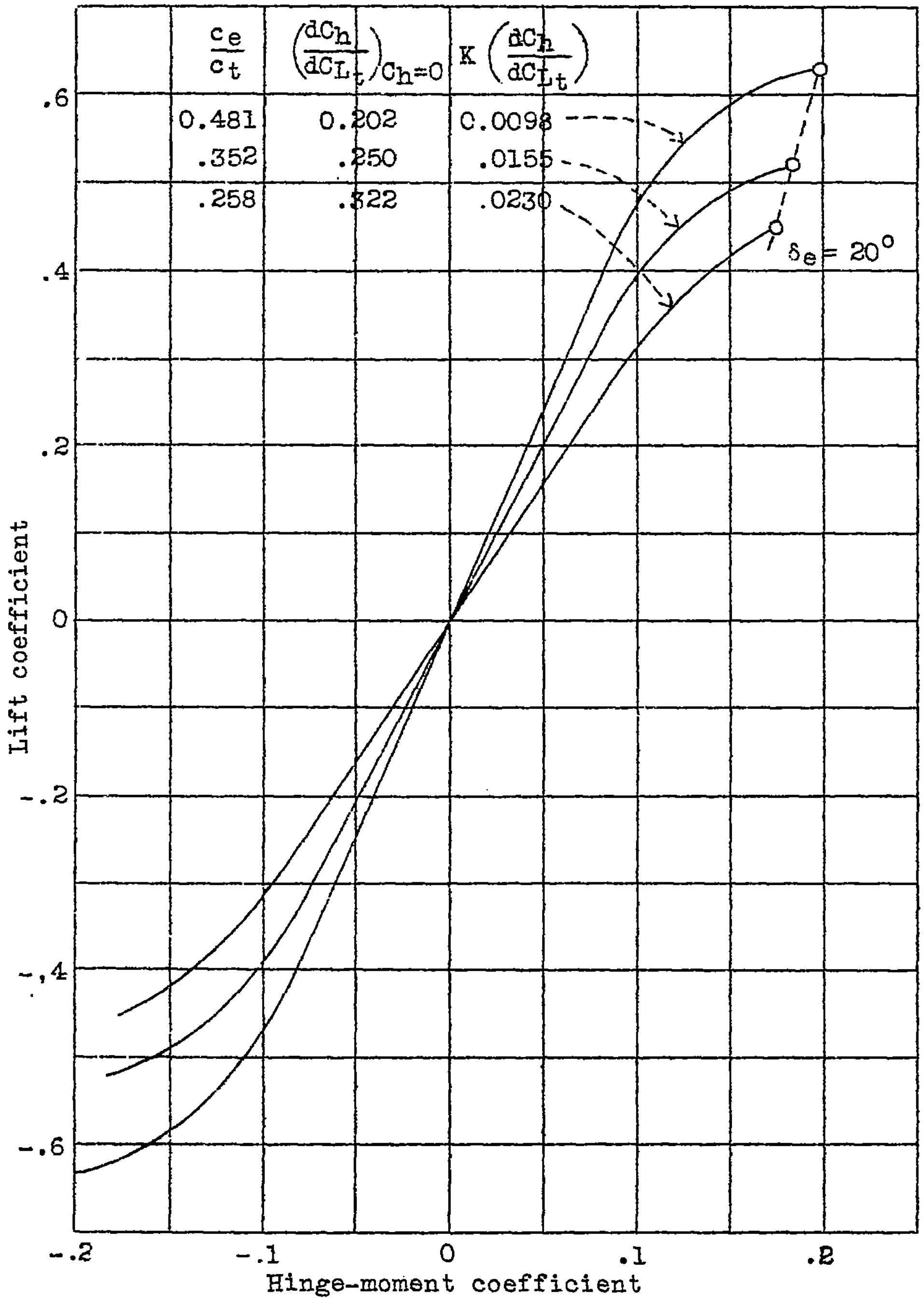


Figure 5.- Effect of ratio of elevator chord to total tail-surface chord on hunge-moment coefficient. (reference 6); aspect ratio, 4.3

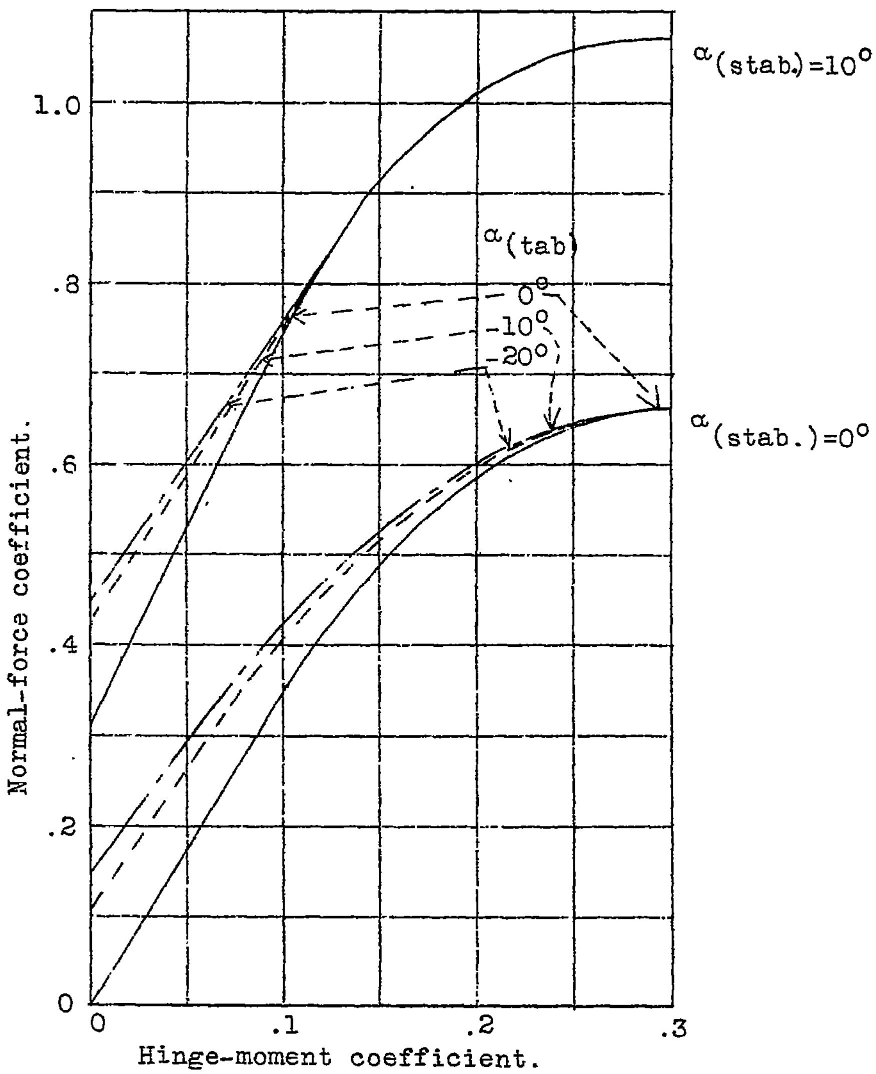


Figure 6.- Effect of trimming tab setting on elevator hinge-moment coefficient (reference 7).